COMPLEX SYSTEMS AND THE MARKETS: A MATHEMATICAL PHYSICIST'S PERSPECTIVE

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Partially based on

- "A dynamical similarity approach to the foundations of coordination and complexity in multiscale systems", University Scholar thesis, University of Connecticut (2003), http://arxiv.org/abs/q-bio/0403026
- "Math, markets and understanding mess", Open (5th June 2009 issue)
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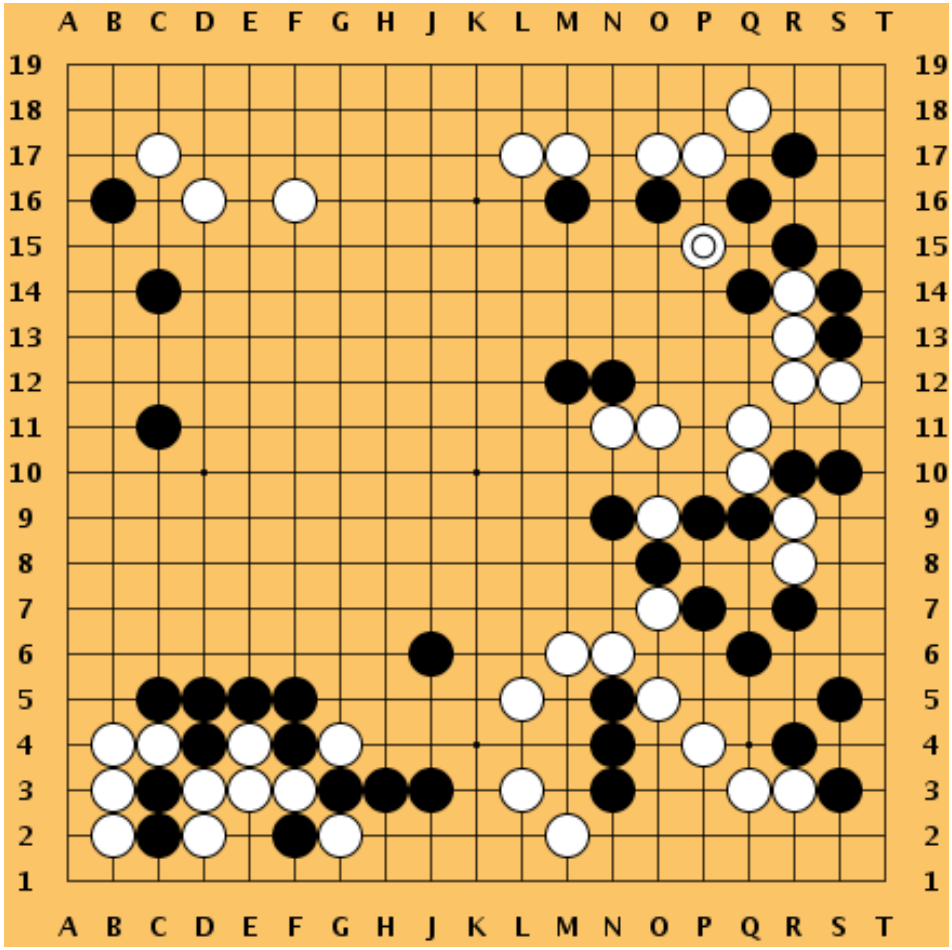
Issues

- From simple to complex- how and when?
- Complex systems and physics- order parameters, universality and phase transitions.
- Relations and models- towards reflexive mathematics.
- Regulations and feedback.
- Some speculations!
- Focus on universality of a large class of physical, biological and economic phenomena!

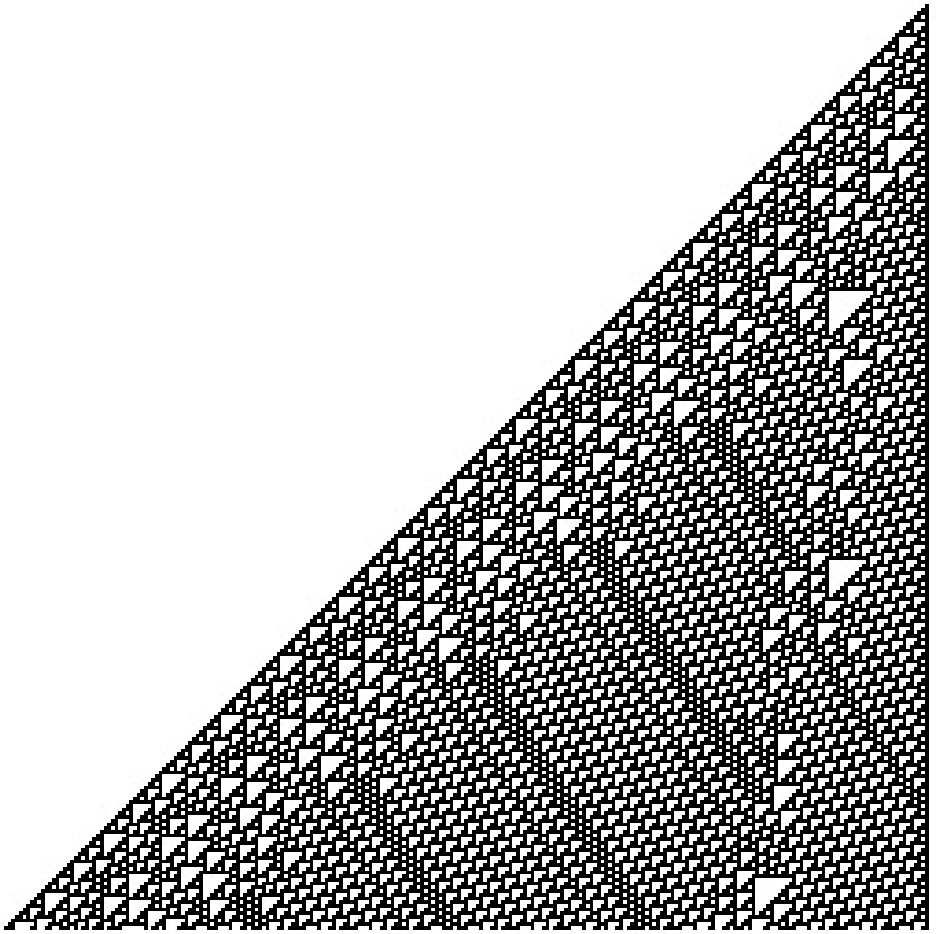
The game of Go



Avalanches

Anthill

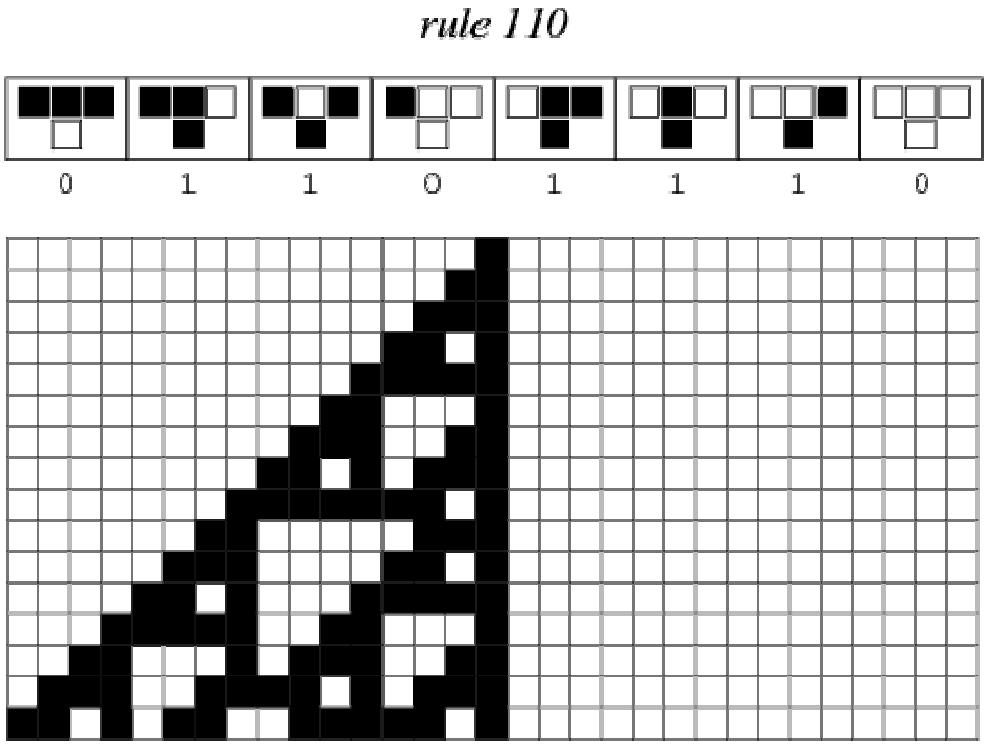
CA Rule 110



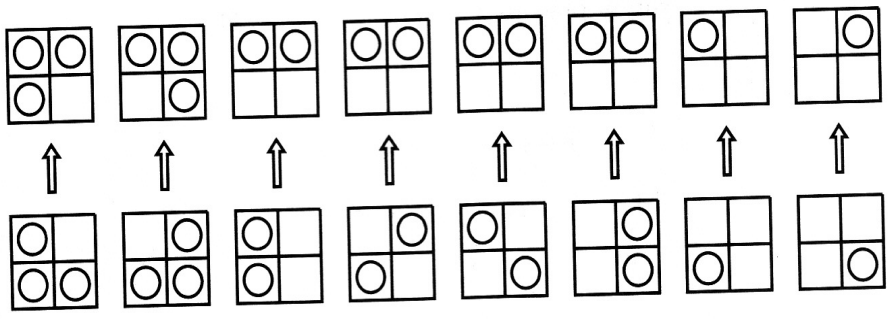
Things in common…

- Simple units, simple laws of combination.
- Extremely complicated behavior.
- Laws governing evolution are “local” in character (more on that later!)
- There is no “global” controlling unit or entity.
- Some of these systems exhibit universal properties, for example the CA rule 110 leads to NP-complete features.

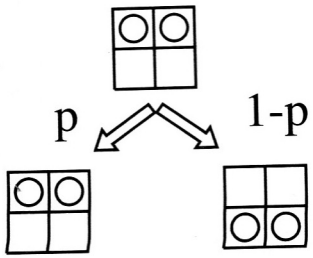
Example: evolution of Rule 110



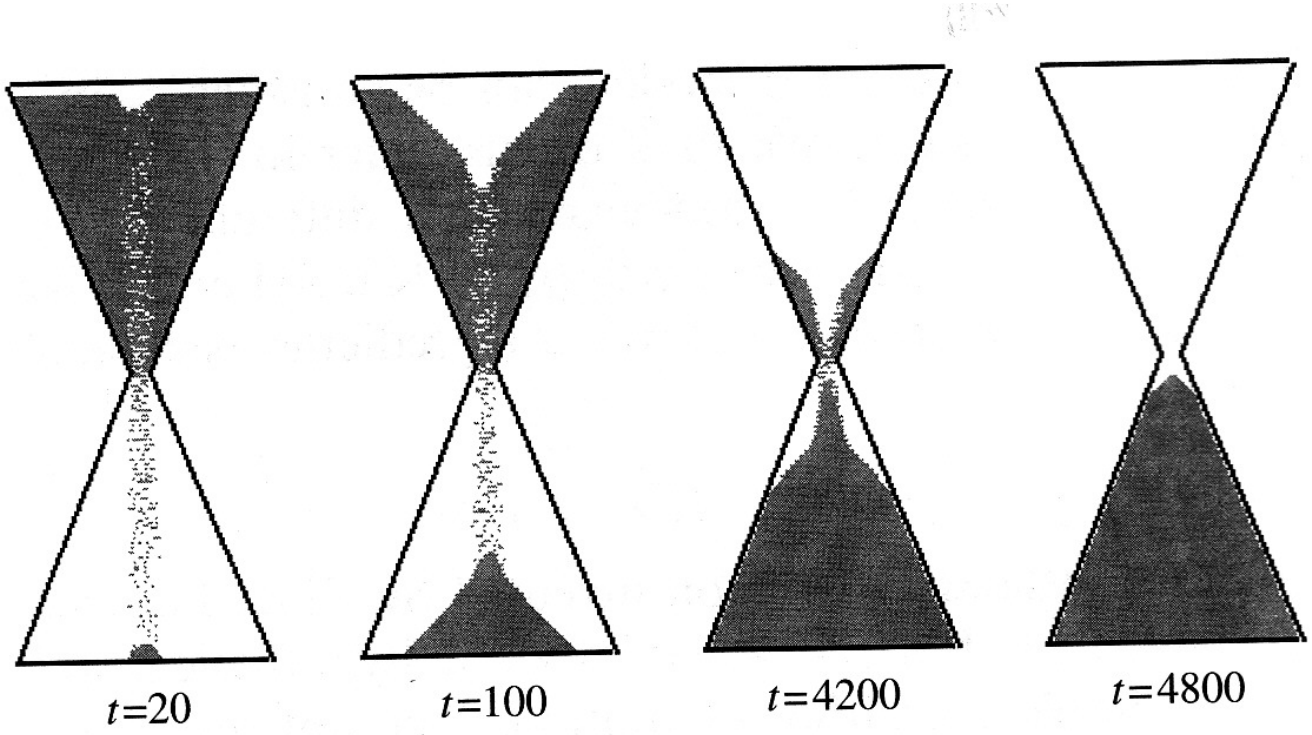
A discrete CA-type model for avalanches (Margolis)



along with a probabilistic feature



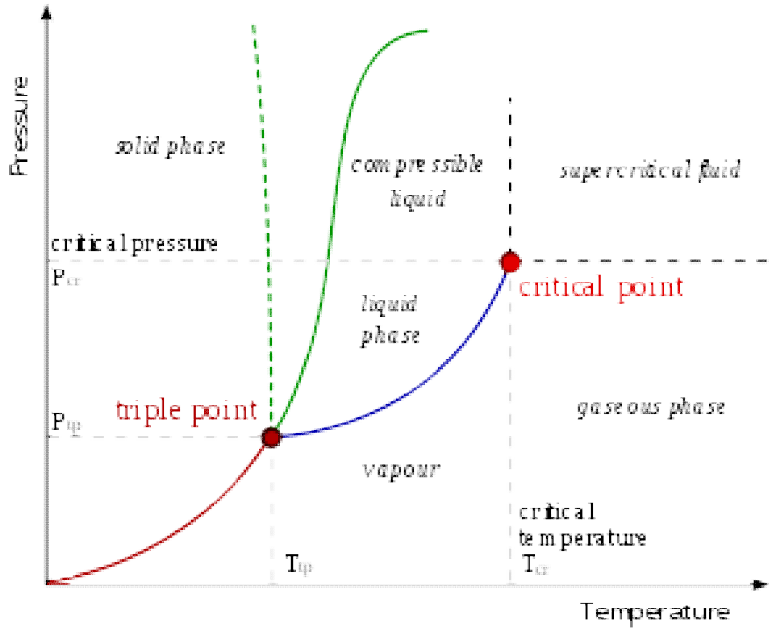
leads to critical phenomena…



Four common features (Kadanoff)

- Universality: Physically different systems show the same behavior.
- Scaling.
- Conformal invariance.
- Renormalization group.
- A canonical example: the Ising model– main object spin $s = \{\pm 1\}$, nontrivial behavior in terms of…

… phase transitions



2nd order phase transitions- irreversible!

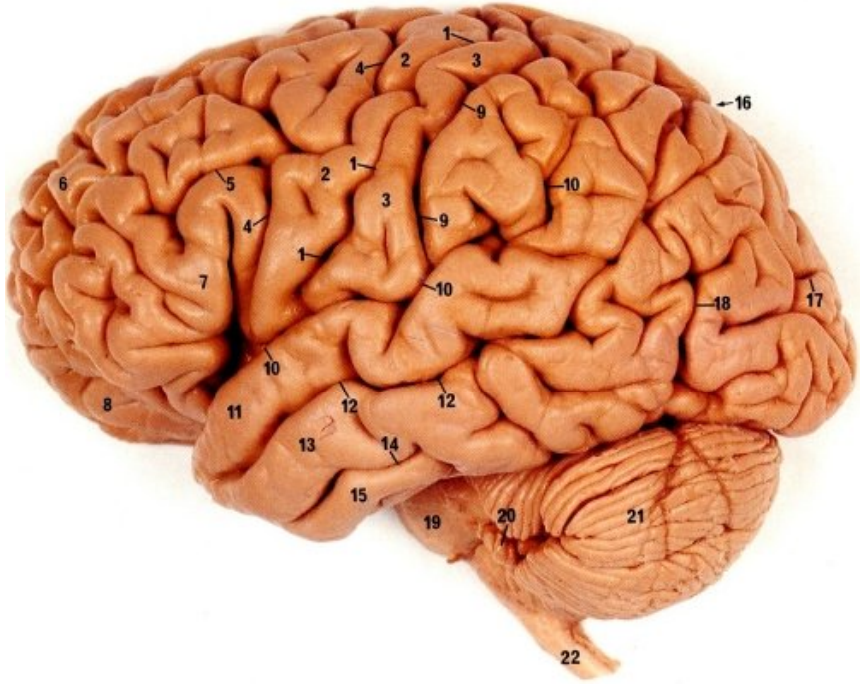
Coarse-graining and thermodynamics

- Consider a box of gas, there are a huge number of degrees of freedom (6 corresponding to each atom multiplied by the no of atoms).
- Yet we can model gases by ignoring the behavior of individual atoms and just looking at averages such as temperature.
- This is called *coarse-graining*, a process by which we can successfully look at the macroscopic averages and ignoring the microscopic dynamics.

This is one of nature's greatest mysteries…

- When you want to build atomic bombs, you study the "macroscopic" theory of atomic nuclei and not be concerned with the constituents of protons in terms of quarks.
- Effectively wash out a huge number of degrees of freedom.
- Can do this for spin systems- instead of looking at a lattice of spins at each vertex, you make blocks of lattice sites.
- Technical note: Hamiltonian scales properly, renormalization group!

Multi-scale and cross-scale dynamics



One hundred billion neurons!

How does neural behavior translates to end effector behavior?

- “large brains gain their powers not so much by the number of neurons they contain, but by the number of scales of neural organization they support” (Alexander & Globus).
- Can for a large class of end effector behavior such as limb movement come up with order parameters that model that behavior without having necessarily understanding how neurons behave.
- We have a PDE for neural behavior- the Hodgkin-Huxley equation- and some for end-effector behavior but there is no obvious way to relate them.

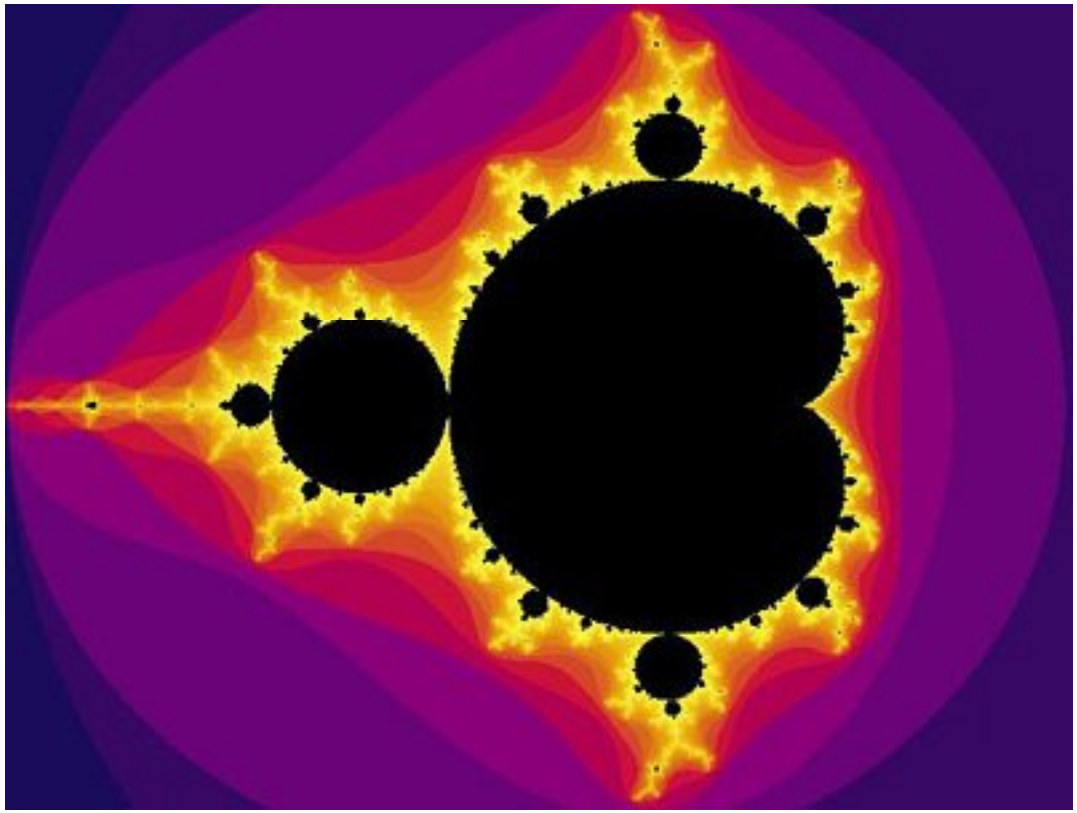
Main issues behind this…

- origin of disorder comes from the observer-theorist's "deliberate refusal to specify (and follow) the locations and behavior of particles on the micro-level that surround a given particle and interact with it" (Mineev-Weinstein)- origin of stochastic terms, systems with noise.
- ". . . complex wholes will usually be developed incrementally over evolutionary time, and that the various intermediate forms must themselves be whole, robust systems . . . " (Clarke)- principle of evolutionary holism.

A theory with many characteristic scales

- "fractal organization" of structure- reflect the truism that all scales of neural organization exploit mechanisms that are structurally embedded in each other and nested together.
- Fractal organization is often reflected in observed/empirical data with distributions with fat-tails as opposed to Gaussian.
- Further explained well in Taleb's *The Black Swan*.

The Mandelbrot set



Predictions of a certain fractal theory of "topological similarity" (Rej, 2003)

- Take a multi-scale system that is capable of utilizing different layer of organization in the performance. Obtain time-series at different scales. Using techniques of phase space reconstruction, one can obtain the geometries from the time-series (Packard et.al., 1980).
- Model the reconstructed flows in the phase spaces thus obtained with low dimensional equations of motion for each scale.
- Claim/conjecture: There exists a suitable affine transformation by which those "reconstructed" equations of motions" transform into one another.

Tying it all up with the matter at hand…

- Complex systems = collection of large number of interlinked units coevolving in time to produce phenomena no single unit could.
- In a complex system it is extremely hard to infer the nature of the units themselves from the behavior of the system as a whole.
- Sensitive dependence on small perturbations (avalanche CA models). Conventional wisdom: system can absorb small shocks, no out-of-equilibrium phenomena. Complex systems: susceptible to small external shocks.

Unpredictability and innovation a la Crutchfield

- "Prediction is difficult, especially of the future"- Neils Bohr.
- Unpredictability is not something that is "one-off"—for most systems it is an intrinsic feature. Complex systems evolve in a way that does not "merely" put a limit on what we *do know about them. The physics of these* systems puts a limit to what we *can know about them.*
- Complex systems have a capacity for innovation. Sometimes such innovation acts in our favor (the boom times) and sometimes against us (pretty much all other times).

“History’s opacity” a la Taleb

- Even if all complex systems— markets, human societies, you name it—evolved according to a single putative evolutionary theory, the lessons would still be unpalatable. Nature can afford to wipe out a species in a rather short period of time yet a couple of more banks collapsing in the current crisis would most probably lead to a complete overhaul of current post WWII financial practices not to mention distress for millions more. as long as economies remain as interlinked and
- In case of a complex system there’s simply no way we can predict precisely the outcomes of a given set of regulations. This is one of consequences of “history’s opacity” (Taleb’s *The Black Swa*n).

Specifics- Strategic Drawing Rights and such

- The business of regulation is also inextricably linked to the theoretical lenses through which one chooses to view the current crisis. For the general equilibrium theorist, instability is always a result of a large external shock. Regulate external shocks away and you'd have a stable system less susceptible to crises as this. For a complexity theorist whose views of the markets are much messier based on the study of simulated behavior of extremely large number of agents working in tandem innovation is intrinsic.

And to sum up…

- The single greatest lesson we have learnt from studying large disparate classes of complex systems— natural, social, economic— is that richness of structure inevitably comes with a price: our inability to predict what lies ahead!

Else who would have guessed …

THE TI

Max 18C, min 5C

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Lehman collapse sends shockwave round world

hares and oil prices plunge, thousands lose jobs

ry Duncan Economics Editor

urs of a global financial meltdown w yesterday as the world's biggest ankruptcy plunged markets into noil.

nvestors were left reeling as the apt demise of the Lehman Broth- investment bank sparked the est shake-up on Wall Street in ades.

nother of US capitalism's biggest tutions, Merrill Lynch, is to be lowed by Bank of America in a billion takeover to save it from se.

ares fell as fear spread through ancial system. Central banks un- urgent measures amid concerns e world economy was entering erous new phase. The Bank of d injected £5 billion of emer- lending into money markets. 5,000 Lehman staff in Britain

are now uestionably in vorst financial s since the t Depression'

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Dow Jones industrial average was down 300 points, or 2.6 per cent. Sentiment was also bolstered by steep falls in oil prices, which dropped by more than $5 a barrel to $96, closing under $100 for the first time in six months and raising hopes that cheaper fuel would ease economic stresses on Western nations.

However, by close of trading the Dow had fallen by more than 500 points — its biggest one-day drop since the reopening after the September 11 attacks — as concerns mounted over the world's largest insurer. Shares in American International Group (AIG), which sponsors Manchester United, fell by 45 per cent after it made an unprecedented approach to the US Federal Reserve for $40 billion in emergency funding.

Last night the Fed asked Goldman Sachs and J P Morgan Chase, two of Wall Street's remaining big banks, to head a $75 billion emergency package to keep AIG afloat.

As central banks battled to stabilise the system, the Fed eased its rules for emergency lending further. It announced that it would accept company shares in return for crisis loans for the first time. In Frankfurt, the European Central Bank injected €30 billion in emergency funds into eurozone markets.

A group of ten global banks also attempted to foster calm, announci