On atoms and space
Why are atoms of space necessary?
Gravity vs quantum physics
Building space through good relations

Building Space

Abhijnan Rej

Max-Planck-Institut für Mathematik (Bonn) and Durham University (Durham)
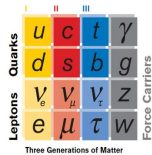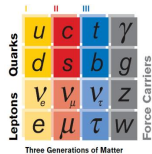
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The Standard Model

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▶ Building blocks of matter = quarks and leptons

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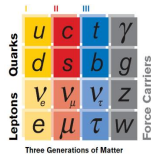
▶ Building blocks of matter = quarks and leptons

▶ All forces (except gravity) mediated by gauge bosons

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Space as habitat of all matter and energy

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Space as habitat of all matter and energy

The Virgo cluster M 91 ($\sim$ 6000 galaxies) is 60 million light years away!

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Space from numbers

Consider the *elliptic curve*

$$y^2 = ax^3 + bx^2 + c$$

Visually, this equation represents (for $a = 1, b = -1, c = 0$ and $a = 1, b = -1, c = 1$)

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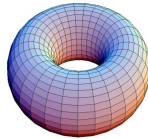Building space through good relations

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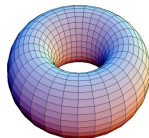$y^2 = x^3 - x$ $y^2 = x^3 - x + 1$

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Considering the same equation but now over *complex numbers* we see the solutions as the torus

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In arithmetic and geometry one is lead to consider much more exotic spaces:

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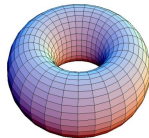**Building space through good relations**

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In arithmetic and geometry one is lead to consider much more exotic spaces:

A Calabi-Yau manifold

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Two principal reasons why we look for *atoms of space*

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▶ Physics: The need for a unified theory of general relativity and quantum physics

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- ▶ Physics: The need for a unified theory of general relativity and quantum physics
- ▶ Arithmetic geometry: The need to treat all invariants of *arithmetic spaces* as once!

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The first problem leads to *quantum gravity*

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I will talk about the first problem today

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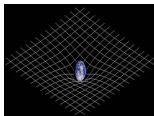Why are atoms of space necessary?
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Two basic equations

Einstein's theory of gravity (general relativity)

$$\mathbf{R} = -\frac{8\pi G}{c^4}\mathbf{T}$$

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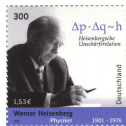
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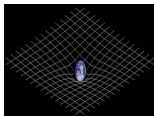
Heisenberg's uncertainty principle for quantum physics

$$\Delta p \Delta x \geq \hbar$$

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Fundamental length and timescales

In order for GR and QP to be good bedfellows, one must have

$$L_p = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-35} m, \; T_p = \sqrt{\frac{\hbar G}{c^5}} \sim 10^{-44} s$$

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Below these scales, its "wild west" for standard physics.

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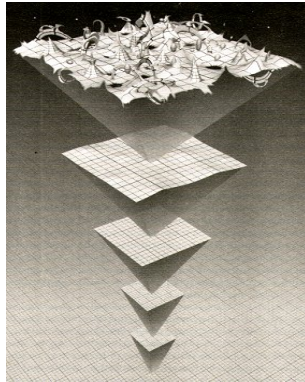**Gravity vs quantum physics**
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Wheeler's quantum foam

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One possible solution to the problem

View spacetime as being tiled by tiles with edgelength $= L_p$

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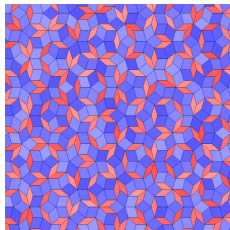**Building space through good relations**
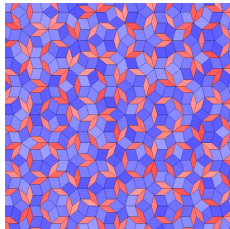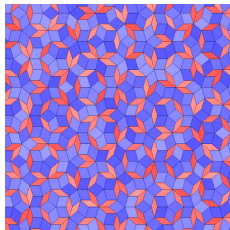
One possible solution to the problem

View spacetime as being tiled by tiles with edgelength $= L_p$

Doesn't work: Nice symmetries of relativity are broken! Causality is violated!

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Penrose's combinatorial approach to spacetime

Consider a network of spins

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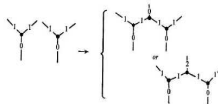Building space through good relations

Penrose's combinatorial approach to spacetime

Consider a network of spins

The edges denote the spacetime path of a particle of spin $=$ integer label $\times \hbar$

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Penrose developed a very precise calculus by which one can join pieces of spin networks according to the rules of quantum mechanics

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A quantum of area

Extending the work of Penrose along with a lot of interesting differential geometry, Rovelli-Smolin gives us the remarkable formula

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Here j_i is the spin of the particle at the edge i.

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Temperature of a blackhole

A breakthrough due to Hawking

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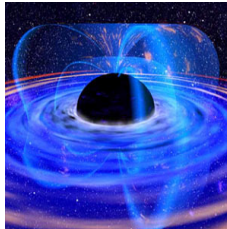
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Black holes are not that black!

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Loop quantum gravity provides an answer for why it is so!

▶ If a blackhole has entropy (or temperature), it must have microstates.

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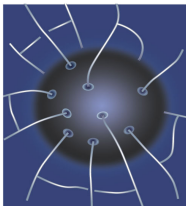
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▶ LQG identifies that these must be related to the quantum of area.

▶ Precise calculations verfies this!

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An artist's conception of the spin foam universe